INVARIANT BUDA - LUND PARTICLE INTERFEROMETRY

T. CSÖRGŐ[1] and B. LÖRSTAD[2]

[1]*MTA KFKI RMKI,*
H-1525 Budapest 114, POB 49, Hungary
E-mail: csorgo@sunserv.kfki.hu

[2]*Physics Department, Lund University,*
S -221 00 Lund, POB 118, Sweden
E-mail: bengt@quark.lu.se

The invariant Buda-Lund parameterization of the two-particle Bose-Einstein correlation functions is presented, its derivation is summarized. In its particular multi-variate Gaussian limiting case, the invariant Buda - Lund parameterization is compared to the Bertsch-Pratt and Yano-Koonin-Podgoretskii parameterizations, advantages and shortcomings are discussed. The invariant Buda-Lund parameterizations are given also for non-Gaussian multi-variate distributions, including damped oscillations in the like-particle correlation function, that are similar to the oscillating intensity correlations of binary stars in stellar interferometry. A separation between the pion and the proton source is also estimated in the Buda - Lund hydro framework, the result is utilized to extract the mean proper time of particle emission with the help of fits to E877 data on non-identical particle correlations by Miskowiec.

1 Introduction

Presently there is an increasing interest both in high energy heavy ion and in high energy particle physics to describe in greater details the space-time picture of particle emission with the help of particle interferometry: the space-time geometry of the freeze-out hypersurface carries information on the existence of a transient Quark-Gluon Plasma phase to be created in heavy ion collisions. Overlapping regions of $(q\overline{q}) + (q\overline{q})$ jets from a decaying W^+, W^- pair at LEP-II may result in unwanted systematic errors in precision determination of W mass in particle physics, and the magnitude of this effect can be estimated with precision only if the space-time picture of these reactions is reconstructed and the magnitude of Bose-Einstein correlations between pions from different W-s is calculated correspondingly.

An invariant formulation of Bose-Einstein correlation functions was found by the Budapest-Lund collaboration in refs. [1,2]. This parameterization is referred as the Buda-Lund parameterization of the correlation function, or BL in short. Although the BL results are rather generic, they contain in particular limiting cases the *power-law*, the *exponential*, the *double-Gaussian*, the *Gaus-*

sian or other parameterizations. The BL parameterization of the correlation function yields not only a *boost-invariant* description, but also a rather *simple* functional form.

We consider high energy heavy ion reactions or single jets in high energy particle physics coming from a fragmentation of an energetic leading quark. In these physical situations, a dominant direction of expansion of the particle source is identified. We denote space-time coordinates by $x = (t, \mathbf{r}) = (t, r_x, r_y, r_z)$ and momentum variables as $k = (E, \mathbf{k}) = (E, k_x, k_y, k_z)$. We choose the direction labelled with subscript z to coincide with the dominant direction of the expansion.

The single-particle spectra and the two-particle correlations are determined in the Wigner-function formalism. As this formalism is fairly well-known now, we move the summary of the derivation to section 3, and jump to the results immediately, as given in the next section. We then compare the BL form of the correlation function to the Bertsch - Pratt (BP) and the Yano-Koonin - Podgoretskii (YKP) 3-dimensional parameterizations in a particular, Gaussian limiting case of the BL correlation function, as the BP and the YKP parameterizations are defined at the moment only for Gaussian correlation functions, as far as we know. We also consider effective separation of sources in the BL hydro model, also described in ref. [1]. The effective separation of like-particle sources is shown to result in small, damped oscillations in the invariant longitudinal component of the two-pion intensity correlation function. The effective separation of unlike-particle sources in the BL hydro parameterization is larger. Such a separation was recently determined experimentally by D. Miskowiec et al in the analysis of E877 data on Au + Au collisions at BNL AGS, based on their estimated $L = 10$ fm/c separation scale and the BL hydro picture we estimate the mean freeze-out time in Au + Au collisions.

2   The Buda-Lund parameterization for Bose-Einstein correlations

The two-particle correlation function is defined as

$$C(\mathbf{k}_1, \mathbf{k}_2) = \frac{N_2(\mathbf{k}_1, \mathbf{k}_2)}{N_1(\mathbf{k}_1)N_2(\mathbf{k}_2)}, \tag{1}$$

a ratio of the two-particle invariant momentum distribution to the product of the single-particle invariant momentum distributions. Hence the correlation function is invariant by definition, see ref. [3] for an exploration of its properties based on its Lorentz invariance. We assume, that with the help of certain experimental methods all non-Bose-Einstein related correlations can be removed from this function, and we focus on the Bose-Einstein correlations only, as

these correlations carry information about the space-time distribution of particle emission. The mean and the relative four-momentum are introduced as

$$K = (k_1 + k_2)/2, \tag{2}$$

$$\Delta k = k_1 - k_2. \tag{3}$$

In the following, the relative momentum four-vector shall be denoted also as $\Delta k = Q = (Q_0, Q_x, Q_y, Q_z) = (Q_0, \mathbf{Q})$, and in general $\mathbf{a}$ shall denote the vector part of a four-vector $a = (a_0, \mathbf{a})$.

A simple invariant formulation of Bose-Einstein correlation functions was given for cylindrically symmetric, longitudinally expanding particle sources in refs.[1,2]. Such sources may be characterized not only with longitudinal but also by transversal and temporal inhomogeneities. In a Gaussian approximation, the Buda-Lund form of the Bose-Einstein correlation function reads as follows:

$$C(\mathbf{\Delta k}, \mathbf{K}) = 1 + \lambda \exp\left(-R_=^2 Q_=^2 - R_\parallel^2 Q_\parallel^2 - R_\perp^2 Q_\perp^2\right), \tag{4}$$

where the fit parameter λ measures a strength of the correlation function. The fit parameter $R_=$ reads as R-timelike, and this variable measures a width of the proper-time distribution. The fit parameter $R_\parallel$ reads as R-parallel, it measures an invariant length parallel to the direction of the expansion. The fit parameter $R_\perp$ reads as R-perpedicular or R-perp. For cylindrically symmetric sources, $R_\perp$ measures a transversal rms radius of the particle emitting source.

The invariant time-like, longitudinal and transverse relative momenta are defined with the help of another, to this point suppressed fit parameter, $\overline{\eta}$, which characterizes the direction of the center of particle emission, $\overline{x} = (\overline{t}, \overline{r}_z)$ in the (t, r_z) longitudinal coordinate space. Such a direction can be characterized by a normalized four - vector $\overline{n}(\overline{x}) = \overline{x}/\overline{\tau}$, where $\overline{n} \cdot \overline{n} = +1$, and $\overline{\tau} = \sqrt{\overline{t}^2 - \overline{r}_z^2}$ is the mean longitudinal proper-time of particle emission. The such a direction-pointing normal vector $\overline{n}$ can be parameterized as $\overline{n} = (\cosh[\overline{\eta}], 0, 0, \sinh[\overline{\eta}])$, where $\overline{\eta} = 0.5 \log\left[(\overline{t} + \overline{r}_z)/(\overline{t} - \overline{r}_z)\right]$ is the space-time rapidity of the center of particle emission, see Fig. 1. Space-time rapidity is a space-time coordinate, that transforms additively in case of longitudinal Lorentz-boosts, similary to the the rapidity variable in momentum space. A boost - invariant decomposition of the relative momenta can be defined, as follows. The invariant time-like, parallel and perp relative momentum components read as

$$Q_= = Q_0 \cosh[\overline{\eta}] - Q_z \sinh[\overline{\eta}] \equiv Q \cdot \overline{n} \tag{5}$$

$$Q_\parallel = Q_0 \sinh[\overline{\eta}] - Q_z \cosh[\overline{\eta}] \equiv Q \times \overline{n}, \tag{6}$$

$$Q_\perp = \sqrt{Q_x^2 + Q_y^2}, \tag{7}$$

$$Q^2 = Q \cdot Q = (Q \cdot \overline{n})^2 - (Q \times \overline{n})^2 - Q_\perp^2. \tag{8}$$

In the above, $a \cdot b = a^\mu b_\mu = a_0 b_0 - \mathbf{ab} = a_0 b_0 - a_x b_x - a_y b_y - a_z b_z$ stands for the inner product of four-vectors. As $\overline{n} \cdot \overline{n} = +1$, this direction is time-like, hence $Q \cdot \overline{n} = Q_=$ is an invariant time-like component of the relative momentum. $Q_\|$ is an invariant relative momentum component in the longitudinal direction (parallel to the beam or to the thrust axis), and $Q_\perp$ is the remaining perp or transverse component of the relative momentum, also invariant for longitudinal boosts.

In short, we have the following 5 *free parameters* for cylindrically symmetric, longitudinally expanding sources: λ, $R_=$, $R_\|$, $R_\perp$ and $\overline{\eta}$. In principle, each of these parameters may depend on the mean momentum $\mathbf{K}$. At any fixed value of the mean momentum $\mathbf{K}$, the 5 free parameters of the invariant Buda-Lund correlation function can be fitted to data; alternalively they can be theoretically evaluated from an assumed shape of the emission function $S(x, k)$, for example see ref[1]. In the Buda-Lund parameterization, the explicit mean momentum dependence of the parameters can be written as follows:

$$C(\mathbf{\Delta k}, \mathbf{K}) = 1 + \lambda(\mathbf{K}) \exp\left(-R_=^2(\mathbf{K})Q_=^2(\mathbf{K}) - R_\|^2(\mathbf{K})Q_\|^2(\mathbf{K}) - R_\perp^2(\mathbf{K})Q_\perp^2\right),$$
(9)

Note that the mean momentum dependence of the relative momentum components $Q_=(\mathbf{K}) = Q_=(n(\mathbf{K}))$ and $Q_\|(\mathbf{K}) = Q_\|(\overline{n}(\mathbf{K}))$ is induced by the mean momentum dependence of the direction pointing normal vector $\overline{n}(\mathbf{K})$, similarly to the dependence of the side and the out components of the relative momentum on the direction of the mean momentum in the Bertsch - Pratt parameterization. Physically, $R_=$, $R_\|$ and $R_\perp$ are longitudinally boost-invariant lengths of homogeneity [4] in the time-like, the longitudinal and the transverse directions. Hence, $R_\|(\mathbf{K})$ is in general less than the total longitudinal extension of the source. Similar statements hold for the other, invariant lengths of homogeneity in the transverse and in the temporal directions, $R_\perp(\mathbf{K})$ and $R_=(\mathbf{K})$.

2.1 Symbolic notation for the side and out components

Up to this point, we assumed a cylindrically symmetric source, where the spatial information about the source distribution in (r_x, r_y) was combined to a single perp radius parameter $R_\perp$.

In order to distinquish easily the zero-th component of the relative momentum Q_0 from the out component of the relative momentum $Q_o = Q_{out}$, $Q_0 \neq Q_o$, we introduce the following symbolic notation in transverse directions:

$$Q_{side} \equiv Q_{..} ,$$
(10)
$$Q_{out} \equiv Q_: ,$$
(11)

4

$$Q_\perp^2 = Q_{side}^2 + Q_{out}^2 = Q_{..}^2 + Q_{:}^2 \ . \tag{12}$$

The idea behind this notation is similar to that of symbolizing the perp direction by index "$\perp$", standing for a direction that is transverse to the longitudinal direction. The longitudinal components of the relative momenta are symbolized by index "$\parallel$", i.e. two vertical parallel lines, imagining that the z axis is in the plane of the paper, pointing upwards, and the considered component is parallel to that direction. The invariant time-like components are indexed by "$=$", not to be confused by the equality sign $=$. This symbol "$=$" is obtained by a 90° rotation of the symbol "$\parallel$". The remaining two orthogonal directions are the "$side$" and the "out", we symbolize them by parallel lines as well. But the two possible orientations of parallel lines in the plane are used up by the symbols $=$ and $\parallel$, hence these two lines are thought to be orthogonal to the plane of the paper, thus they are symbolized by two dots. The side component is described by putting the dots side-by-side, which yields index "$..$". The remaining out direction is orthogonal to all the above directions, we rotate side by 90° to obtain the symbol "$:$". This symbolic notation stands for the longitudinally boost-invariant decomposition of the relative momenta. Similarly, the side and the out radii can be denoted as

$$R_{side} = R_{..}, \tag{13}$$
$$R_{out} = R_{:}. \tag{14}$$

Cross-terms, if any, can be denoted by straightforward mixing of the symbols, e.g. a possible side-out cross-term may be denoted by "$. :$", an out-long cross-term by "$: |$" and a side-long cross-term by "$..|$" etc. In a general Gaussian form, suitable for studying opacity effects, the Buda-Lund invariant BECF can be denoted as

$$C(Q,K) = 1 + \lambda_* \exp\left(-R_=^2 Q_=^2 - R_\parallel^2 Q_\parallel^2 - R_{..}^2 Q_{..}^2 - R_:^2 Q_:^2\right) \tag{15}$$

Note, that this equation is identical to eq. (44) of ref. [1], rewritten into the new, symbolic notation of the Lorentz-invariant directional decomposition.

The above equation may be relevant for a study of expanding shells as well as opacity effects as recently suggested by H. Heiselberg [9]. The lack of transparency in the source may result in an effective source function, that looks like a crescent in the side-out reference frame [9]. The overall cylindrical symmetry of particle production is maintained for simultaneous rotations in the x and the k space, but the emission function $S(x,K)$ at a given fixed direction of the mean transverse momentum $K_\perp = (0, K_x, K_y, 0)$ has different width in the *out* direction and another, possibly larger width in the *side* direction [1].

5

3 General derivation of Buda-Lund shape – not only for Gaussians

First we define the correlation function with the help of the Wigner-function formalism, then we introduce the intercept parameter λ_* in the core-halo picture. Then we evaluate the correlation function in terms of longitudinally boost-invariant variables and we end up with a general Buda-Lund form of the correlation function.

3.1 Wigner Function Formalism

The two-particle inclusive correlation function is defined and approximately expressed in the Wigner function formalism as

$$C(\Delta k; K) = \frac{N_2(\mathbf{k}_1, \mathbf{k}_2)}{N_1(\mathbf{k}_1)\, N_1(\mathbf{k}_2)} \simeq 1 + \frac{|\, \tilde{S}(\Delta k, K)\, |^2}{|\, \tilde{S}(0, K)\, |^2}. \tag{16}$$

In the above line, the Wigner-function formalism[10,11,12] is utilized assuming fully chaotic (thermalized) particle emission. The covariant Wigner-transform of the source density matrix, $S(x, k)$ is a quantum-mechanical analogue of the classical probability that a boson is produced at a given $x^\mu = (t, \mathbf{r}) = (t, r_x, r_y, r_z)$ with $k^\mu = (E, \mathbf{k}) = (E, k_x, k_y, k_z)$. The auxiliary quantity

$$\tilde{S}(\Delta k, K) \;=\; \int d^4 x S(x, K) \exp(i\Delta k \cdot x)$$

appears in the definition of the BECF, with $\Delta k = k_1 - k_2$ and $K = (k_1 + k_2)/2$. The single- and two-particle inclusive momentum distributions (IMD-s) are defined in terms of the cross-section, they can be evaluated as

$$N_1(\mathbf{k}) = \frac{E}{\sigma_t}\frac{d\sigma}{d\mathbf{k}} = \tilde{S}(\Delta k = 0, k) \quad \text{and} \quad N_2(\mathbf{k_1}, \mathbf{k_2}) = \frac{E_1 E_2}{\sigma_t}\frac{d\sigma}{d\mathbf{k}_1\, d\mathbf{k}_2}, \tag{17}$$

where σ_t is the total inelastic cross-section. Note that in this work we utilize the absolute normalization of the emission function: $\int \frac{d^3\mathbf{k}}{E} d^4 x\, S(x, k) = \langle n \rangle$.

It has been shown recently in refs. [13,16,15,17] that for explicitely symmetrized n-particle system of bosons with variable number of bosons, the two-particle Bose-Einstein correlation function is properly defined by eq. (16).

3.2 Effects from Large Halo of Long-Lived Resonances

If the bosons originate from a core which is surrounded by a halo of long-lived resonances, the IMD and the BECF can be calculated in a straightforward

manner. The detailed description is given in refs.[18,19], here we recapitulate only the basic idea along the lines of ref. [18]. *If* the emission function can be approximately divided into two parts, representing the core and the halo, $S(x; K) = S_c(x; K) + S_h(x; K)$ and *if* the halo is characterized by large length-scales so that $\tilde{S}_h(Q_{min}; K) << \tilde{S}_c(Q_{min}; K)$ at a finite experimental resolution of $Q_{min} \geq 10$ MeV, then

$$N_1(\mathbf{k}) = N_{1,c}(\mathbf{k}) + N_{1,h}(\mathbf{k}), \tag{18}$$

$$C(\Delta k; K) = 1 + \lambda_* \frac{|\tilde{S}_c(\Delta k, K)|^2}{|\tilde{S}_c(0, K)|^2}, \tag{19}$$

where $N_{1,i}(\mathbf{k})$ stands for the IMD of the halo or core for $i = h, c$ and

$$\lambda_* = \lambda_*(K = k) = \left[\frac{N_{1,c}(\mathbf{k})}{N_1(\mathbf{k})}\right]^2. \tag{20}$$

The phenomenological λ_* parameter has been introduced to the literature by Deutschmann long time ago[20]. In the core/halo picture, this effective intercept parameter $\lambda(\mathbf{k})$ *can be interpreted* as the *momentum dependent* square of the ratio of the IMD of the core to the IMD of all particles emitted, assuming completely incoherent emission from the source.

The validity of the core/halo picture to any given reaction is not guaranteed, hence systematic checks of the applicability of this simplifying picture has to be performed, similarly to the ones done in ref. [22].

3.3 Invariant, generic decomposition of the Bose-Einstein correlation function to a Buda - Lund form

For systems expanding relativistically in one direction (r_z), it is advantageous to introduce the longitudinally boost invariant variable τ and the space-time rapidity η that transforms additively under longitudinal boosts,

$$\tau = \sqrt{t^2 - r_z^2}, \tag{21}$$

$$\eta = 0.5 \log\left[(t + r_z)/(t - r_z)\right]. \tag{22}$$

Similarly, in momentum space one introduces the transverse mass m_t and the rapidity y as

$$m_t = \sqrt{E^2 - p_z^2}, \tag{23}$$

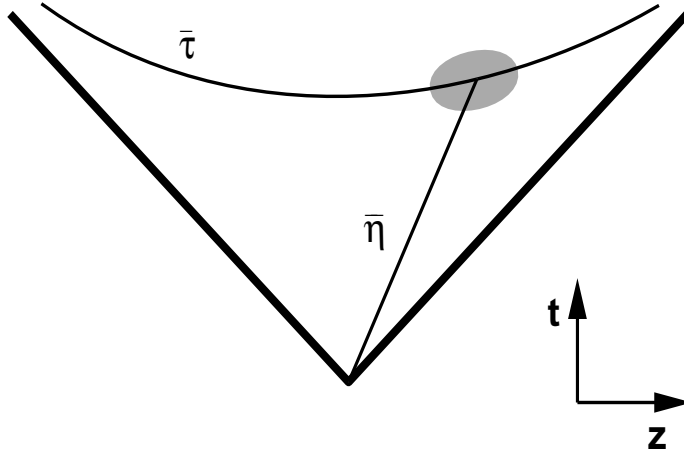$$y = 0.5 \log\left[(E + p_z)/(E - p_z)\right]. \tag{24}$$

Fig. 1. Space-time picture particle emission for a given fixed mean
momentum of the pair. The mean value of the proper-time and the
space-time rapidity distributions is denoted by $\overline{\tau}$ and $\overline{\eta}$. As the rapidity
of the produced particles changes from the target rapidity to the projec-
tile rapidity the $(\overline{\tau}(y), \overline{\eta}(y)$ variables scan the surface of mean particle
production in the (t, r_z) plane.

In order to obtain (at least approximately) a boost-invariant picture, we
characterize the source of particles in the boost invariant variables τ, m_t and
$\eta - y$. For systems that are only approximately boost-invariant, the emission
function may also depend on the deviation from mid-rapidity y_0. The scale
on which the approximate boost-invariance breaks down is denoted by $\Delta\eta$, a
parameter is related to the width of the rapidity distribution.

A simple generalization to non-Gaussian correlators in the Buda-Lund pic-
ture is obtained if we assume that the emission function factorizes as a product
of an effective proper-time distribution, a space-time rapidity distribution and
a transverse coordinate distribution [14,1] as

$$S_c(x, K)d^4x = H_*(\tau)G_*(\eta)I_*(r_x, r_y)\, d\tau\, \overline{\tau}d\eta dr_x dr_y. \tag{25}$$

In the above, subscript $*$ stands for a dependence on the mean momentum,
the mid-rapidity and the scale of violation boost-invariance $(K, y_0, \Delta\eta)$. The
function $H_*(\tau)$ stands for an effective proper-time distribution (that includes
an extra factor τ from the Jacobian $d^4x = d\tau\, \tau\, d\eta, dr_x dr_y$, in order to simplify
the results). The effective (K dependent) space-time rapidity distribution is
denoted by $G_*(\eta)$, while the effective transverse distribution is denoted by
$I_*(r_x, r_y)$. In the above equation, the mean proper-time $\overline{\tau}$ is factored out to

8

keep the dimensionless nature of the distribution functions. Such a pattern of particle production is visualized on Fig. 1.

In case of hydrodynamical models, as well as in case of a decaying Lund strings[14,2], production of particles with a given momentum rapidity y is limited to a narrow region in space-time around $\overline{\eta}$ and $\overline{\tau}$. If the sizes of such an effective source are sufficiently small (or with other words if the Bose-Einstein correlation function is sufficiently broad), the plane-waves that induce the Fourier - transformation in the correlation function can be decomposed in the shaded region on Fig. 1 as follows:

$$\exp[i\Delta E(t_1 - t_2) - iQ_z(r_{z,1} - r_{z,2})] \simeq \exp[iQ_=(\tau_1 - \tau_2) - iQ_\|\overline{\tau}(\eta_1 - \eta_2)], \quad (26)$$

$$\exp[-iQ_x(r_{x,1} - r_{x,2}) - iQ_y(r_{y,1} - r_{u,2})] = $$
$$= \exp[-iQ_:(r_{:,1} - r_{:,2}) - iQ_{..}(r_{..,1} - r_{..,2})].(27)$$

With the help of this small source size (or large relative momentum) expansion, the two-particle Bose-Einstein correlation function can be written into the following Buda-Lund form:

$$C(\Delta k, K) = 1 + \lambda_*(K)\frac{|\tilde{H}_*(Q_=)|^2}{|\tilde{H}_*(0)|^2}\frac{|\tilde{G}_*(Q_\|)|^2}{|\tilde{G}_*(0)|^2}\frac{|\tilde{I}_*(Q_:, Q_{..})|^2}{|\tilde{I}_*(0;0)|^2}. \quad (28)$$

This is the factorized Buda-Lund invariant decomposition of the BE correlation function; the resulting Fourier-transformed proper-time , space-time rapidity and transversal coordinate distributions can be of power-law, exponential, Gaussian or other types, corresponding to the underlying space-time structure of the particle emitting source:

$$\tilde{H}_*(Q_=) = \int_0^\infty d\tau \exp(iQ_=\tau)H_*(\tau), \quad (29)$$

$$\tilde{G}_*(Q_\|) = \int_{-\infty}^\infty d\eta \exp(-iQ_\|\overline{\tau}\eta)G_*(\eta), \quad (30)$$

$$\tilde{I}_*(Q_:, Q_{..}) = \int_{-\infty}^\infty d\tau \exp(-iQ_:r_: - iQ_{..}r_{..})I_*(r_:, r_{..}). \quad (31)$$

What are the fit parameters in eq. (28)? It is clear that apart from the shape parameters of the proper-time, space-time rapidity and the transverse distribution of the production points two additional space-time parameters enter the fit: the mean proper-time of the particle production, $\overline{\tau}$ in eq. (30) and the angular direction $\overline{\eta}$ that enters the definition of $Q_=$ and $Q_\|$ in eqs. (5),6). In turn,

the parameters $(\overline{\tau}, \overline{\eta})$ are measurable from the detailed analysis of the multi-dimensional Bose-Einstein correlation functions for any value of the rapidity and m_t of the particle pair. The total longitudinal region corresponds to the region in (t, r_z) where particles of arbitrary momenta are emitted from. This region can be reconstructed in the Buda-Lund formalism, by determining $(\overline{\tau}, \overline{\eta})$ and the widths of $H_*(\tau)$ and $G_*(\eta)$ at various fixed values of the momentum of the particles, reproducing the shaded region of Fig. 1 for each fixed value of the mean momentum of the pair. Such shaded regions are the same as local maps in cartography. If the momentum distribution of the produced particles is integrated over, the overlapping shaded regions are combined to a global picture of particle emission in space-time, similarly the way how local maps can be combined to an atlas in cartography by gradually displacing the centers of the local maps but keeping an overlap region between the neighbouring local pictures.
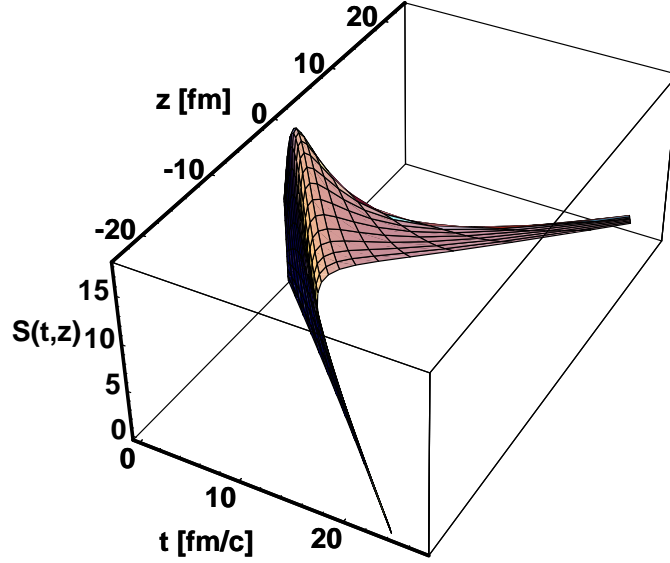

Fig. 2. Space-time picture of particle emission as reconstructed from a combined analysis [5,6,1] of particle correlations and spectra in $h + p$ reactions at CERN SPS in (t, r_z) plane. The momentum of the emitted particles is integrated over, from ref. [30].

10

This programme has been carried out first by the NA22 collaboration in a Gaussian approximation, keeping the means and the variances only of the proper-time distribution, the space-time rapidity distribution and the transverse distribution coordinates of particle production. A locally thermalized, longitudinally expanding medium (possibly corresponding to the vacuum filled by the sea of virtual quarks, anti-quarks and gluons) was found to be in a good agreement with the NA22 particle spectra and correlations, despite the fact that the mean multiplicity of the produced charged particles is only $\langle n \rangle = 8$. See R. Hakobyan's contribution to this conference proceedings for the reconstructed space-time picture as well as for greater details.

A similar reconstruction of the longitudinal space-time structure of particle production in 200 AGeV $S + Pb$ reactions is reported in the contribution of Ster to this volume.

For even further general possibilities about the structure of particle emission, see refs. [1,14,2].

4 Earlier, Gaussian parameterizations of BE Correlations

We briefly summarize here the Bersch-Pratt and the Yano-Koonin parameterization of the Bose - Einstein correlation functions, to point out some of their advantages as well as draw-backs and to form a basis for comparision.

4.1 The Bertsch-Pratt parameterization

The Bertsch-Pratt (BP) parameterization of Bose-Einstein correlation functions is one of the oldest, widely used parameterization called also as side-out-longitudinal decomposition [23,24] of the correlation function.

This directional decomposition was originally devised for extracting the contribution of the long duration of particles from a decaying Quark-Gluon Plasma, as happens in the mixture of a hadronic and a QGP phase if the rehadronization phase transition is a strong first order transition.

The BP parameterization in a compact form reads as

$$C(\mathbf{\Delta k}, \mathbf{K}) = 1 + \lambda \exp\left[-R_s^2 Q_s^2 - R_o^2 Q_o^2 - R_l^2 Q_l^2 - 2R_{ol}^2 Q_l Q_o\right]. \quad (32)$$

Here index o stands for *out* (and not the temporal direction), s for *side* and l for *longitudinal*. In a more detailed form, the mean momentum dependence of the various components are explicitly shown:

$$C(\mathbf{\Delta k}, \mathbf{K}) = 1 + \lambda(\mathbf{K}) \exp\left[-R_s^2(\mathbf{K})Q_s^2(\mathbf{K}) - R_o^2(\mathbf{K})Q_o^2(\mathbf{K})\right. $$
$$\left. -R_l^2(\mathbf{K})Q_l^2 - 2R_{ol}^2(\mathbf{K})Q_l Q_o(\mathbf{K})\right], \quad (33)$$

11

where the mean and the relative momenta are defined as

$$\mathbf{K} = 0.5(\mathbf{k}_1 + \mathbf{k}_2), \tag{34}$$

$$\mathbf{\Delta k} = \mathbf{k}_1 - \mathbf{k}_2, \tag{35}$$

$$Q_l = k_{z,1} - k_{z,2}, \tag{36}$$

$$Q_o = Q_o(\mathbf{K}) = \mathbf{\Delta K} \cdot \mathbf{K}/|\mathbf{K}|, \tag{37}$$

$$Q_s = Q_s(\mathbf{K}) = |\mathbf{\Delta k} \times \mathbf{K}|/|\mathbf{K}|. \tag{38}$$

It is emphasized that the BP radius parameters are measuring lengths of homogeneity, and in general characterize that region of space-time that emits particles with a given mean momentum $\mathbf{K}$. Not only the radius parameters but also the decomposition of the relative momentum to the side and the out components depends on the mean momentum $\mathbf{K}$.

In arbitrary frame, Gaussian radius parameters can be defined, and sometimes they are also referred to BP radii, when the spatial components of the relative momentum vector are taken as independent variables. It should be emphasized, that the BP radii that contain a *well-defined mixture* of the longitudinal, temporal and transverse invariant radii. However, the BP radius parameters themselves are not invariant, they depend on the frame where they are evaluated [1], reflecting space-time variances [7,8] of the *core* [22] of the particle emission.

$$C^{c/h}(\mathbf{k}, \mathbf{\Delta k}) = 1 + \lambda_*(\mathbf{K}) \, \exp\left(-R_{i,j}^2(\mathbf{K})\mathbf{\Delta k}_i \mathbf{\Delta k}_j\right), \tag{39}$$

$$\lambda_*(\mathbf{K}) = [N_{\mathbf{c}}(\mathbf{K})/N(\mathbf{K})]^2, \tag{40}$$

$$R_{i,j}^2(\mathbf{K}) = \langle (x_i - \beta_i t)(x_j - \beta_j t) \rangle_{\mathbf{c}} - \langle (x_i - \beta_i t) \rangle_{\mathbf{c}} \langle (x_j - \beta_j t) \rangle_{\mathbf{c}}, \tag{41}$$

$$\langle f(x, \mathbf{k}) \rangle_{\mathbf{c}} = \int d^4 x f(x, \mathbf{k}) S_{\mathbf{c}}(x, \mathbf{k})/\int d^4 x S_{\mathbf{c}}(x, \mathbf{k}), \tag{42}$$

where $S_{\mathbf{c}}(x, \mathbf{k})$ is the emission function that characterizes the central core. Note, that the tails of the emission function are dominated by the halo of long-lived resonancess $S_h(x, \mathbf{k})$ and even a small admixture of e.g. η and η' mesons increases drastically the space-time variances of particle production, and makes the interpretation of the BP radii in terms of the total emission function $S = S_c + S_h$ unreliable, as pointed out in ref. [22].

It was shown already in ref. [14] that the duration of the particle emission contributes predominantly to the out direction, if the Longitudinal Center of Mass System (LCMS, ref. [14]) is selected for the determination of the radius parameters. In LCMS, the mean momentum of the pair has no longitudinal component. In this frame, the BP radii have a particularly simple form, if the

coupling between the r_x and the t coordinates is also negligible, $\langle \tilde{r}_x \tilde{t} \rangle = \langle \tilde{r}_x \rangle \langle \tilde{t} \rangle$:

$$R_{out}^2(\mathbf{K}) = \langle \tilde{r}_x^2 \rangle_{\mathbf{c}} - \beta_t^2 \langle \tilde{t}^2 \rangle_{\mathbf{c}} \tag{43}$$

$$R_{side}^2(\mathbf{K}) = \langle \tilde{r}_y^2 \rangle_{\mathbf{c}} \tag{44}$$

$$R_{long}^2(\mathbf{K}) = \langle \tilde{r}_z^2 \rangle_{\mathbf{c}} \tag{45}$$

$$R_{out,long}^2(\mathbf{K}) = \langle \tilde{r}_z(\tilde{r}_x - \beta_t \tilde{t}) \rangle_{\mathbf{c}}, \tag{46}$$

where $\tilde{x} = x - \langle x \rangle$. From the above, the advantage of the LCMS frame is clear: In this LCMS frame, information on the temporal scale couples *only* to the out direction and it enters both the out radius component and the out-long cross-term. Note that for cylindrically symmetric sources other possible cross-terms, e.g. the side-out or the side-long cross terms were shown to vanish in a Gaussian approximation, due to symmetry reasons [7,8].

An advantage of the BP parameterization is that there are no kinematic constraints between the side, out and long components of the relative momenta, hence the BP radii are not too difficult to determine experimentally.

4.2   The Yano-Koonin-Podgoretskii parameterization

A covariant parameterization has been worked out for non-expanding sources by Yano, Koonin and Podgoretskii (YKP) [25,26]. This parameterization was recently applied to expanding sources by the Regensburg group [27,28], by allowing the YKP radius and velocity parameters be momentum dependent. This parameterization reads as

$$C(\mathbf{Q}, \mathbf{K}) = 1 + \exp\left[-R_\perp^2 Q_\perp^2 - R_\parallel^2(Q_l^2 - Q_0^2) - \left(R_0^2 + R_\parallel^2 \right)(Q \cdot U)^2 \right], \tag{47}$$

(Note that in YKP index $_0$ refers to the time-like components). When the momentum dependence of the YKP radii is explicitly shown, this reads as

$$C(\mathbf{Q}, \mathbf{K}) = 1 + \exp\left[-R_\perp^2(\mathbf{K}) Q_\perp^2 - R_\parallel^2(\mathbf{K})(Q_z^2 - Q_0^2) \right.$$
$$\left. - \left(R_0^2(\mathbf{K}) + R_\parallel^2(\mathbf{K}) \right)(Q \cdot U(\mathbf{K}))^2 \right], \tag{48}$$

where the fit parameter $U(\mathbf{K})$ is interpreted [27,28] as a *four-velocity* of a fluid-element [32]. This YKP parameterization is introduced to create a diagonal Gaussian expression in the "rest frame of the fluid-element".

This form has an advantage as compared to the BP parameterization, namely that the three extracted YKP radius parameters, $R_\perp$, $R_\parallel$ and R_0 are invariant, independent of the frame where the analysis is performed, while U^μ transforms as a four-vector.

The practical price one has to pay for this advantage is that the kinematic region in the Q_0, Q_l, $Q_\perp$ space, where the parameters can be fitted may become rather small. This follows from the unequalities $Q_{inv}^2 \geq 0$ and $Q_0^2 \geq 0$, which yields

$$0 \leq Q_0^2 \leq Q_z^2 + Q_\perp^2, \tag{49}$$

and the narrowing of the regions in $Q_0^2 - Q_z^2$ with decreasing $Q_\perp$ makes the experimental determination of the YKP parameters difficult, especially when the analysis is performed far away from the LCMS rapidities [or more precisely from the frame where $U^\mu = (1, 0, 0, 0)$]. Hence in practice the YKP parameters can be well determined in the LCMS frame, where the longitudinal component of the U is generally small. But in the LCMS, the interpretation of the BP radii is also simple, similarly to that of the YKP radii.

Theoretical problems with the YKP parameterization are explained below. *a)* The YKP radii contain components proportional to $\frac{1}{\beta_t}$, that lead to divergent terms for particles with very low p_t [27,28]. *b)* The YKP radii are not even defined for all Gaussian sources [27,28]. Especially, for opaque sources or for expaning shells with $\langle \tilde{r}_x^2 \rangle < \langle \tilde{r}_y^2 \rangle$ the algebraic relations defining the YKP "velocity" parameter become ill-defined and result in imaginary values of the YKP "velocity",[27,28]. *c)* The YKP "velocity" is defined in terms of space-time variances at fixed mean momentum of the particle pairs [27,28]. Thus, for expanding systems, the proper interpretation of the parameter U is *not* a flow velocity of a source element, as thought before, but a combination of space-time variances of the source at a fixed mean momentum **K** . (Note, however, that for static, non-expanding sources the interpretation of U^μ as the velocity of a Gaussian source can be preserved corresponding to refs. [25,26]).

In kinetic theory that provides the basis for hydrodynamics, the flow velocity can be locally defined as a weighted average of particle *momenta*, all particles being in the same cell in coordinate space. The local flow velocity $u^\mu(x)$ hence becomes a function of the position x but the momentum of the particles was averaged out, hence $u^\mu(x)$ is formally independent of the momentum. The four-current is defined [29] as an average over the local momentum distribution

$$j^\mu(x) = \int \frac{d^3 k}{k^0} k^\mu f(x, k) \tag{50}$$

where $f(x, k)$ stands for the phase-space distribution function. The local flow velocity can be defined with this current as the unit vector proportional to the current:

$$u^\mu(x) = j^\mu(x) / \sqrt{j^\mu(x) j_\mu(x)}. \tag{51}$$

On the other hand, the YKP parameter U corresponds to a weighted average of particle *coordinates*, all particles being characterized with the momentum **K**. Hence in general $U(\mathbf{K}) \neq u(x)$. The local velocity of particles at a fixed momentum is independent of the density distribution of particle production in coordinate-space. In fact, the only four-velocity that can be uniquely assigned to a set of particles each with momentum **K** is simply $u_{\mathbf{K}}^{\mu} = (E_{\mathbf{K}}/m, \mathbf{K}/m)$, where m stands for the mass of the particles. Hence the YKP parameter $U(\mathbf{K})$ should not be interpreted as a mean *flow* velocity of a fluid element that emits particles with momentum **K**, at least not in the well-defined sense of kinetic theory.

4.3 Comments on the hydro model parameterization

The Buda-Lund hydro parameterization is recapitulated in the contributions of Ster and Hakobyan [31,30]. The main idea behind the Buda-Lund parameterization is to characterize the hydro fields (temperature profile, transversal flow, density distribution) with their means and variances only. Similar models are studied by the Regensburg [7,8,28] as well as the Kiev groups [33].

Note that $\overline{n}^{\mu}(\mathbf{K})$ normal-vector rests in the (t, r_z) plane for the Buda-Lund parameterization. On the other hand, the $U^{\mu}(\mathbf{K})$ YKP velocity was thought to be some sort of flow velocity, that characterizes some local momentum distribution. Hence, $\overline{n}^{\mu}$ and $U(\mathbf{K})$ are defined in *different spaces*: in the coordinate space and in the momentum space, respectively. In case of the Buda-Lund form, the coordinate-space interpretation of $\overline{n}$ is needed to obtain the expansion of the plane-wave $\exp(i\Delta k \cdot \Delta x)$ in eq. (26), which is essential in expressing the correlation function in terms of Fourier-transformed proper-time distributions and space-time rapidity distributions in eq. (28). If the space-time interpretation of the Buda-Lund direction $\overline{n}(\mathbf{K})$ is lost, it becomes impossible to reconstruct the space-time picture of particle emission for systems with strong longitudinal expansion. That could be the reason why such a reconstruction was not yet achieved by the NA49 experiment, that applied the YKP parameterization. However, the space-time picture of longitudinally expanding particle emitting sources was reconstructed from the detailed fitting to NA22 and NA44 particle correlations and spectra with the help of the Buda-Lund parameterization, properly preserving the interpetation of of $\overline{n}(\mathbf{K})$ as a spatial direction. See Fig. 2, and the contributions of Hakobyan, Ster and Seyboth to this conference proceedings [30,31,32].

5 Separation of particle sources in Buda-Lund type hydro models

We discuss some results relating the separation of effective sources for identical and non-identical particles, even if both kind of particles appear from the same system that is assumed to be in local thermal equilibrium.

5.1 Separation of effective sources for non-identical particles

Recently, Lednicky and collaborators suggested to study non-identical particle-correlations, to learn which particles are emitted earlier and which particles were emitted later [34]. The analysis of E877 data resulted in an effective separation of pion and proton sources at forward rapidities at the AGS [35].

Such separation of pion and proton source in space-time occurs as a natural result in the Buda-Lund hydro models and its various re-incarnations and improved modifications, because heavier particles are more frozen to the flow than the lighter ones.

In the Buda-Lund parameterization, we have a K dependent normalvector, pointing towards the center of the particle production.

In the LCMS, ref [14] , we find

$$n = n(\overline{x}(\mathbf{K})) = (\cosh[\overline{\eta}], 0, 0, \sinh[\overline{\eta}]) \tag{52}$$

$$\overline{\eta} = \overline{\eta}(\mathbf{K}) = \frac{y_0 - y}{1 + \Delta\eta^2 m_t/T_0}, \tag{53}$$

where T_0 is the central freeze-out temperature at the mean freeze-out time, and $\Delta\eta$ characterizes the finite longitudinal size of the expanding hydro source in space-time rapidities.

For non-identical particle correlations [34], e.g. π and p, the velocities of the particles must be similar.

If $v_\pi = v_p$, then we have $m_t^\pi << m_t^p$. For pions and protons with the same velocity, using $m_\pi = 140$ MeV, $m_p = 1$ GeV, $T = 140$ MeV and $\Delta\eta^2 = 2$ the Buda-Lund model yields:

$$\overline{\eta}^p - \overline{\eta}^\pi \simeq \frac{y - y_0}{3} \tag{54}$$

where y_0 is the mid-rapidity, y is the rapidity of the pion and the proton, and 3 is the numerically estimated coefficient.

The spatial separation between protons and pions is about $L \simeq 10$ fm/c at $y - y_0 \simeq 2$ [35]. This can be used to estimate the mean freeze-out proper-time of particle production as $\tau_s = L/(\eta_s^p - \eta_s^\pi) \simeq 15$ fm/c.

5.2 Example for a non-Gaussian Buda-Lund correlation function: separation of the effective source for identical pions

In the Buda-Lund type hydro models, the emission function $S(x,k)$ is written as a product of the local phase-space density $f(x,k)$ and a Cooper-Frye pre-factor $d^4\Sigma^\mu(x)k_\mu$ that yields the flux of particles through the freeze-out hypersurface, or through a distribution of freeze-out hypersurfaces[1]:

$$S(x,k)d^4x = f(x,k)\,d^4\Sigma \cdot k \tag{55}$$

$$f(x,k) = \frac{g}{(2\pi)^3} \frac{1}{\exp\left[\dfrac{k\cdot u(x) - \mu(x)}{T(x)}\right] + s} \tag{56}$$

$$d^4\Sigma(x)\cdot k = m_t \cosh[\eta - y]H_*(\tau)\,d\tau\,\overline{\tau}d\eta\,dr_x\,dr_y \tag{57}$$

where g stands for the degeneracy factor, and $s = -1$, 0, $+1$ for Bose, Boltzmann or Fermi statistics, and an approximate boost-invariant shape of the freeze-out hypersurface distribution is assumed. Using the exponential form of the $\cosh[\eta - y]$ factor, the effective emission function $S_*(x,k) = \frac{\tau}{\overline{\tau}}S(x,k)$ can be written as a sum of two components:

$$S(x,k) = 0.5[S_+(x,k) + S_-(x,k)], \tag{58}$$

$$S_+(x,k) = m_t \exp[+\eta - y]H_*(\tau)f(x,k), \tag{59}$$

$$S_-(x,k) = m_t \exp[-\eta + y]H_*(\tau)f(x,k). \tag{60}$$

These effective emission function components are subject to Fourier - transformation in the Buda-Lund approach. In an improved saddle-point approximation, the two components $S_+(x,k)$ and $S_-(x,k)$ can be Fourier - transformed independently, finding the separate maxima (saddle point) $\overline{x}_+$ and $\overline{x}_-$ of $S_+(x,k)$ and $S_-(x,k)$, and repeating the saddle-point calculation for the two components separately. As a result, one gains the following non - Gaussian correlation function from the Buda-Lund hydro model specified in ref.[1]

$$C(Q_=, Q_\|, Q_.., Q_:) = 1 + \lambda_* \,\Omega(Q_\|)\,\exp(-Q_\|^2 R_\|^2 - Q_=^2 R_=^2 - Q_\perp^2 R_\perp^2), \tag{61}$$

$$\Omega(Q_\|) = [\cos^2(Q_\| R_\| \Delta\overline{\eta}) + \sin^2(Q_\| R_\| \Delta\overline{\eta})\tanh^2(\overline{\eta})], \tag{62}$$

$$\frac{1}{\Delta\overline{\eta}^2} = \frac{1}{\Delta\eta^2} + \frac{m_t}{T_0}\cosh[\overline{\eta}]. \tag{63}$$

This result goes beyond the single Gaussian version of the saddle-point calculations of ref.[7,8]. This results goes also beyond the results obtainable in the YKP or the BP parameterizations. In principle, the improved saddle-point

calculation gives more accurate analytic results than the numerical evaluation of space-time variances, as it keeps more information on the shape of the correlation function.

Although the above result is non-Gaussian, because the factor $\Omega(Q_\parallel)$ results in oscillations of the correlator, the result is still explicitly boost-invariant.

Note that the oscillations are typically small and the Gaussian remains a good approximation to eq. (61), but with modified radius parameters.

The oscillations are due to a possible separation of the pion source to two components, due to an identical splitting of the Cooper-Frye pre-factor or the flux term in the emission function. We obtain two separate effective sources that create oscillations in the intensity correlation function, similarly to the oscillations in the intensity correlations of photons from binary stars in stellar astronomy. However, these oscillations in high energy physics are much smaller, as the effective separation between the particle sources for identical pions, $\overline{\eta}_+ - \overline{\eta}_-$ smaller, than $\Delta\overline{\eta}$, the width of the $G_{*+}(\eta_+)$ and the $G_{*-}(\eta_-)$ distributions. In stellar astronomy, the separation between the binary stars is much larger than the diameter of the stars, hence the intensity oscillations in the two-photon correlation function are stronger then in the Buda-Lund type hydro models.

6 Highlights:

The invariant Buda-Lund notation is introduced to describe Bose-Einstein correlation functions. This invariant notation scheme yields simple expressions not only for Gaussian but also for non-Gaussian expanding sources as well. It seems that the Buda-Lund form is the simplest and the most compact characterization of the two-particle Bose-Einstein correlation functions for relativistic, expaning systems.

With the help of the Buda-Lund formulation, and a combined analysis of particle correlations and spectra [1] , the space-time picture of the particle production in the longitudinal (t, r_z) plane can be reconstructed. Examples for such a reconstruction are shown in refs. [31,30].

We pointed out how to estimate the mean freeze-out time using non-identical particle correlations data, the Buda-Lund hydro model and the natural separation of forward moving protons and pions in Buda-Lund type hydro models.

Finally we have shown how Buda-Lund type hydro models can be rewritten to an effective, two-components sources, by splitting the flux terms. We found small damped oscillations in the intensity correlation function, reminiscent to

the intensity correlations of photons from binary stars.

Acknowledgments

References

1. T. Csörgő and B. Lörstad, hep-ph/9509213, Phys. Rev. **C54** (1996) 1390
2. T. Csörgő and B. Lörstad, hep-ph/9511404, Proc. XXV-th ISMPD (World Scientific, D. Bruncko et al, 1996) p. 661
3. S. E. Voloshin, W. E. Cleland, Phys. Rev. **C54** (1996) 3212
4. A. Makhlin and Y. Sinyukov, Z. Phys. **C39**, (1988) 69
   A. Makhlin and Yu. Sinyukov, Sov. J. Nucl. Phys. **46** (1987) 354, Yad. Phys. **46** (1987) 637;
   Yu. M. Sinyukov, Nucl. Phys. A566 (1995) 589c.
5. T. Csörgő, B. Lörstad and J. Zimányi, Phys. Lett. **B338** (1994) 134-140
6. T. Csörgő, Phys. Lett. **B347** (1995) 354-360
7. S. Chapman, P. Scotto and U. Heinz, Phys. Rev. Lett. **74** (1995) 4400
8. S. Chapman, P. Scotto and U. Heinz, Heavy Ion Physics **1** (1995) 1
9. H. Heiselberg, nucl-th/9809077
   H. Heiselberg, nucl-th/9609022
10. S. Pratt, T. Csörgő, J. Zimányi, Phys. Rev. **C42** (1990) 2646
11. W. A. Zajc, in NATO ASI Series **B303**, p. 435
12. S. Chapman and U. Heinz, Phys. Lett. **B340** (1994) 250
13. D. Miskowiec and S. Voloshin, nucl-ex/9704006
14. T. Csörgő and S. Pratt, Report No. KFKI-1991-28/A p. 75
15. T. Csörgő and J. Zimányi, Phys. Rev. Lett. **80** (1998) 916-918,
    T. Csörgő and J. Zimányi, hep-ph/9811283
16. J. Zimányi and T. Csörgő, hep-ph/9705432 - v.3
17. Q. H. Zhang, hep-ph/9807257,
    Q. H. Zhang, hep-ph/9805499,
    Q. H. Zhang, P. Scotto and U. Henz, nucl-th/9805046
18. T. Csörgő, B. Lörstad, and J. Zimányi,
    hep-ph/9411307, Z. Phys. C **71** (1996) 491

19. J. Bolz, U. Ornik, M. Plümer, B.R. Schlei, and R.M. Weiner,
    Phys. Lett. B **300**, 404 (1993),
    J. Bolz, U. Ornik, M. Plümer, B.R. Schlei, and R.M. Weiner,
    Phys. Rev. D **47**, 3860 (1993).
20. M. Deutschmann et al, Nucl. Phys. **B204** (1982) 333
21. T. Csörgő, hep-ph/9705422, Phys. Lett. B **409** (1997) 11
22. S. Nickerson, T. Csörgő and D. Kiang, nucl-th/9712059,
    Phys. Rev. C57 (1998) 3251 - 3262
23. G. F. Bertsch, Nucl. Phys. **A498** (1989) 173c
    G. F. Bertsch and G. E. Brown, Phys. Rev. **C40** (1989) 1830
24. S. Pratt, Phys. Rev. **D33** (1986) 1314
25. F. Yano and S. Koonin, Phys. Lett. **B78** (1978) 556
26. M. I. Podgoretskii, Sov. J. Nucl. Phys. **37** (1983) 272
27. Y.-F. Wu, U. Heinz, B. Tomasik, U. A. Weideman,
    Eur. Phys. J. **C1** (1998) 599 - 617
28. B. Tomasik and U. Heinz, nucl-th/9707001,
    Eur. Phys. J. **C4** (1998) 327 - 338
29. S. de Groot, W. van Leeuwen, C. van Weert, Relativistic Kinetic Theory
    (North Holland, Amsterdam, 1980).
30. N. M. Agababyan et al, EHS - NA22 Collaboration,
    Phys. Lett. **422** (1998) 359,
    R. Hakobyan for the NA22/EHS collaboration, in proc. of CF'98
    (Mátraháza, Hungary, June 14-21, 1998, World Scientific, Singapore,
    1999, ed. T. Csörgő, S. Hegyi, R. C. Hwa and G. Jancsó, this volume).
31. A. Ster et al, in proc. CF'98 (Mátraháza, Hungary, June 14-21, 1998,
    World Scientific, Singapore, 1999, ed. T. Csörgő, S. Hegyi, R. C. Hwa
    and G. Jancsó, this volume).
32. P. Seyboth et al, in proc. CF'98 (Mátraháza, Hungary, June 14-21, 1998,
    World Scientific, Singapore, 1999, ed. T. Csörgő, S. Hegyi, R. C. Hwa
    and G. Jancsó, this volume).
33. S. V. Akkelin and Yu. M. Sinyukov, preprint ITP - 63 -94E
    S. V. Akkelin and Yu. M. Sinyukov, Phys. Lett. **B356** (1995) 525
    S. V. Akkelin and Yu. M. Sinyukov, Z. Phys. **C72** (1996) 501
    Yu. M. Sinyukov, S. V. Akkelin and N. Xu, nucl-th/9807030.
34. R. Lednicky et al, Phys. Lett. **B373** (1996) 30
    S. Voloshin, R. Lednicky, S. Panitkin and N. Xu,
    Phys. Rev. Lett. **79** (1997) 4766
35. D. Miskowiec, nucl-ex/9809003, in proc. CRIS'98, June 8-12, 1998, Aci-
    castello, Italy (World Scientific, Singapore, 1999 in press)